G-2/234/21

Roll No. ....

M.Sc. II Semester Examination, 2021

MATHEMATICS

Paper V

(Advanced Discrete Mathematics-II)

Time : 3 Hours] [Max. Marks : 80

Note : *All questions are compulsory. Question Paper comprises of 3 sections. Section A is objective type/multiple choice questions with no internal choice. Section B is short answer type with internal choice. Section C is long answer type with internal choice.*

SECTION A **1×10=10**

(Objective Type Questions)

Note : *Choose the one correct answer :*

1. The maximum number of edges in a simple graph with n vertices is :

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$

(c) $\frac{n^2}{2}$ (d) $\frac{(n^2-1)}{2}$

2. A complete bipartite graph $K_{m,n}$ are Euler graphs if :

(a) m and n both are even

(b) m and n both are odd

P.T.O.

(c) m odd and n even

(d) m even and n odd.

3. A graph is said to be tree if :

(a) It is connected

(b) It has n vertices and $(n-1)$ edges

(c) It is minimally connected

(d) It has all above properties

4. A spanning tree of a graph is a tree which contains :

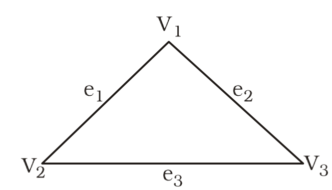
(a) All the edges of the graph

(b) All the vertices of the graph

(c) Some vertices of the graph

(d) Some vertices and some edges

5. The adjancency matrix of the given graph is :



(a) $\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array}\begin{array}{c} e_1\ e_2\ e_3 \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$

(b) $\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array}\begin{array}{c} e_1\ e_2\ e_3 \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$

(c) $\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array}\begin{array}{c} e_1\ e_2\ e_3 \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$

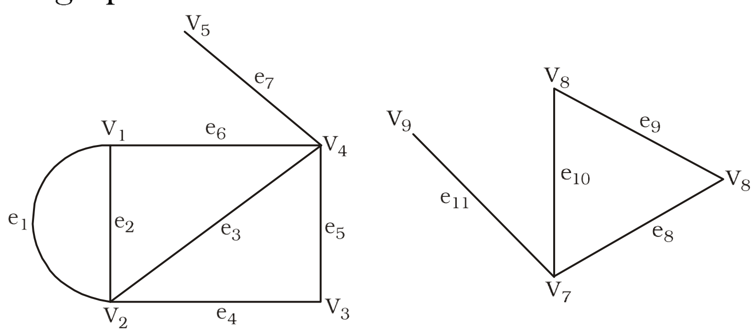
(d) None of these

6. If G is a connected planer graph having e edges and v vertices where $u \geq 3$, select the correct option :

(a) $v \leq 3e - 6$ (b) $e \leq 3u - 6$

(c) $e \leq 2v - 6$ (d) None of these

7. Rank and nullity of the following disconnected graph :



(a) rank = 7
nullity = 5

(b) rank = 5
nullity = 7

(c) rank = 7
nullity = 4

(b) none of these

8. Two states are called 0-equivalent if :

(a) Both states have input and same output

(b) Both states have input

(c) Both states have same output

(d) Next state is 1-equivalent

9. In Moore machine choose the correct option :

(a) The output function depends on present state and current input

(b) The output function depends only current input

(c) The output function depends only present state

(d) None of these

10. The arrow indicate in a finite state machine :

(a) accepting state (b) initial state

(c) rejecting state (d) none of these

SECTION B **5×4=20**

(Short Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit-I

1. Show that the maximum number of edges in a complete bipartite graph of n vertices is $\frac{n^2}{4}$.

Or

What is the maximum number of vertices in a graph with 35 edges and all vertices are of degree at least 3.

Unit-II

2. Define cut sets of a graph and show that every cut set in a connected graph G contains at least one branch of every spanning tree of G.

Or

Explain the path matrix of a graph. Give an example.

Unit-III

3. Show that every connected graph with n vertices and $(n - 1)$ edges is a tree.

Or

Explain Binary search tree.

Unit-IV

4. Explain a non-deterministic finite automation and give one example.

Or

Explain finite state machines and their transition table and diagrams.

Unit-V

5. Find a deterministic acceptor equivalent to $M = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$ δ is given in table :

State table

States/Σ	a	b
$\rightarrow q_0$	q_0, q_1	q_2
q_1	q_0	q_1
q_2	—	q_0, q_1

Or

Explain the Turing Machine.

SECTION C **10×5=50**

(Long Answer Type Questions)

Note : *Attempt one question from each unit.*

Unit-I

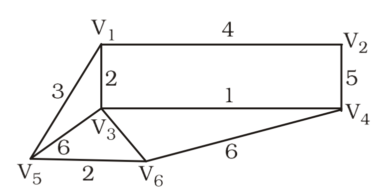
1. Let G be a simple graph with n vertices. If G has k components, then show that maximum number of edges that G can have $\frac{(n-k)(n-k+1)}{2}$.

Or

Let G be a connected planar graph with v vertices and e edges and let r be the number of regions in a planar representation of G. Then show that $v - e + r = 2$.

Unit-II

2. Explain the Kruskal algorithm and find the minimal spanning tree for the graph

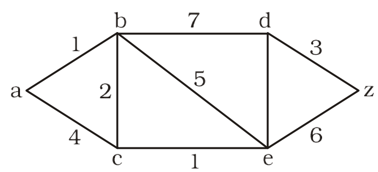


Or

A tree has $2n$ vertices of degree 1, $3n$ vertices of degree 2 and n vertices of degree 3. Determine the number of vertices and edges in the tree.

Unit-III

3. Apply Disjkstra algorithm to find the shortest path from a to z in the graph given below :



Or

Explain the tree traversals and give example for each one.

Unit-IV

4. For the finite state machine shown below :

State	**Input**		**Output**
	0	1	
⇒ *A*	*F*	*B*	0
B	*D*	*C*	0
C	*G*	*B*	0
D	*E*	*A*	1
E	*D*	*A*	0
F	*A*	*G*	1
G	*C*	*H*	1
H	*A*	*H*	1

(a) List all 0-equivalent states.

(b) Find all equivalent states and obtain an equivalent finite state machine with the smallest number of states.

Or

Minimize the machine whose state table is given below :

State	**Input**		**Output**
	0	1	
⇒ S_0	S_3	S_6	1
S_1	S_4	S_2	0
S_2	S_4	S_1	0
S_3	S_2	S_0	1
S_4	S_5	S_0	1
S_5	S_3	S_5	0
S_6	S_4	S_2	1

Unit-V

5. Consider the Moore machine describe by the transition table. Construct the corresponding Mealy machine :

Moore Machine

Present state	**Next State** $a = 0$	$a = 1$	**Output**
$\to q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

Or

Consider the Mealy machine described by the given transition table. Constuct a Moore machine which is equivalent to the Mealy machine.

Mealy Machine

Present State	**Next State** Input $a = 0$ State	Output	Input $a = 1$ State	Output
$\to q_1$	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	0
q_4	q_4	1	q_3	0

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